WEEKLY TEST TYJ -1 TEST - 28 R
SOLUTION Date 24-11-2019

[PHYSICS]

1. The weight of the aircraft is balanced by the upward force due to the Pressure difference.

i.e.,

$$\Delta P = \frac{mg}{A} = \frac{(4\times 10^5 \text{ kg})(10 \text{ ms}^{-2})}{500 \text{ m}^2} = \frac{4}{5}\times 10^4 \text{ N m}^{-2}$$

$$= 8\times 10^3 \text{ N m}^{-2}$$

Let v_1, v_2 are the speed of air on the lower and upper surface of the wings of the aircraft and P_1, P_2 are the pressures there.

Using Bernoulli's theorem, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}(\rho v_2^2 - \rho v_1^2)$$

$$\Delta P = \frac{\rho}{2}(v_2 + v_1)(v_2 - v_1)$$

or $$v_2 - v_1 = \frac{\Delta P}{\rho v_{av}}$$

Here, $$v_{av} = \frac{v_1 + v_2}{2} = 720 \text{ km h}^{-1}$$

$$= 720\times\frac{5}{18}\text{ms}^{-1} = 200 \text{ ms}^{-1}$$

$$\therefore \quad \frac{v_2 - v_1}{v_{av}} = \frac{\Delta P}{\rho v_{av}^2} = \frac{\frac{4}{5}\times 10^4}{1.2\times(200)^2}$$

$$= \frac{4\times 10^4}{5\times 1.2\times 4\times 10^4} = 0.17$$

2. Total cross-sectional area of the femurs is,

$A = 2 \times 10\ \text{cm}^2 = 2 \times 10 \times 10^{-4}\ \text{m}^2 = 20 \times 10^{-4}\ \text{m}^2$

Force acting on them is

$F = mg = 40\ \text{kg} \times 10\ \text{ms}^{-2} = 400\ \text{N}$

∴ Average pressure sustained by them is

$$P = \frac{F}{A} = \frac{400\ \text{N}}{20 \times 10^{-4}\ \text{m}^2} = 2 \times 10^5\ \text{N m}^{-2}$$

3. Since pressure is transmitted undiminished throughout the water

$$\therefore \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

where F_1 and F_2 are the forces on the smaller and on the larger pistons respectively and A_1 and A_2 are the respective areas.

$$\therefore \quad F_2 = \frac{A_2}{A_1}F_1 = \frac{\pi(D_2/2)^2}{\pi(D_1/2)^2}F_1\left(\frac{D_2}{D_1}\right)^2 F_1$$

$$= \frac{(3 \times 10^{-2}\ \text{m})^2}{(1 \times 10^{-2}\ \text{m})^2} \times 10\ \text{N} = 90\ \text{N}$$

4. The scent sprayer is based on Bernoulli's theorem.

5. From Archemedes' principle, this apparent loss in weight is equal to the weight of the liquid displaced by the body.

Also, volume of candle = Area × length

$$= \pi\left(\frac{d}{2}\right)^2 \times 2L$$

Weight of candle = Weight of liquid displaced

$$V\rho g = V'\rho' g'$$

$$\Rightarrow \quad \left(\pi\frac{d^2}{4}\times 2L\right)\rho = \left(\pi\frac{d^2}{4}\times L\right)\rho'$$

$$\Rightarrow \quad \frac{\rho}{\rho'} = \frac{1}{2}$$

Since candle is burning at the rate of 2 cm/h, then after an hour, candle length is $2L - 2$

$$\therefore \quad (2L-2)\rho = (L-x)\rho'$$

$$\therefore \quad \frac{\rho}{\rho'} = \frac{L-x}{2(L-1)}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{L-x}{2(L-1)}$$

$$\Rightarrow \quad x = 1 \text{ cm}$$

Hence, in one hour it melts 1 cm and so it falls at the rate of 1 cm/h.

6. According to Bernoulli's principle

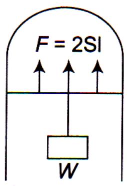
$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

At the sides the velocity is higher, so the pressure is lower. But the pressure at a given horizontal level must be equal, therefore the liquid rises at the sides to some height to compensate for this drop in pressure.

7. Because film tries to cover minimum surface area.

8. Here, $W = 1.5 \times 10^{-2}$ N,

$l = 30 \text{ cm} = 30 \times 10^{-2}$ m



A liquid film has two free surfaces. A slider will support the weight when the force of surface tension action upwards on the slider ($2Sl$) balances the downward force due to weight ($= W$)

9. Energy needed = Increment in surface energy

= (surface energy of n small drops) – (surface energy of one big drop)

$= n4\pi r^2 T - 4\pi R^2 T = 4\pi T(nr^2 - R^2)$

10. The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

11. The thin ring is in contact with water from both inside and outside. So, contact length is $2 \times 20 = 40$ cm

$$F_{\min} = F_{ST} + W = (75 \times 10^{-5}) \times 40 + 0.1 = 0.130 \text{ N}$$

12. It may be noted that the soap film has two free surfaces. So, the effective length is 8ℓ.

13. Effective area $= 2 \times 0.02 \text{ m}^2 = 0.04 \text{ m}^2$

Surface energy $= 5 \text{ m}^{-1} \times 0.04 \text{ m}^2 = 2 \times 10^{-1}$ J

14. $W = [2 \times 4\pi(3r)^2 - 2 \times 4\pi r^2]\, T = 64\, \pi r^2 T$

15. $F = 2\pi r_1 T + F = 2\pi r_2 T$

$$= 2\pi(r_1 + r_2)T$$

$$= 2 \times 3.14(10 + 5)(72) = 6782.4 \text{ dyne}$$

16. $h = \dfrac{2\sigma \cos\theta}{r\rho g}$ or $r = \dfrac{2\sigma \cos\theta}{h\rho g}$

or $r = \dfrac{2 \times 75 \times 10^{-3} \times \cos 0°}{3 \times 10^{-2} \times 10^3 \times 10} \text{m} = 5 \times 10^{-4} \text{ m}$

17. $h = h_0 = \dfrac{2T \cos\theta}{\rho g r}$

$$= \frac{2(72)\cos 0°}{(1)(1000)\left(\dfrac{1}{40}\right)} = 57.6 \text{ cm}$$

Since ℓ (= 50 cm) $< h_0$.

$h = 50$ cm

18. Excess pressure inside the air bubble $= \dfrac{2T}{r}$

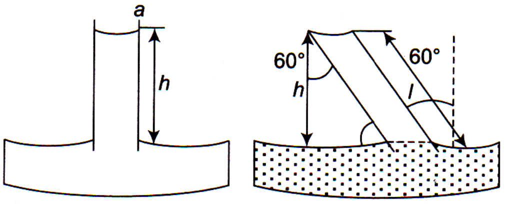
$\Rightarrow \quad P_{in} - P_{out} = \dfrac{2T}{r} = \dfrac{2 \times 70 \times 10^{-3}}{0.1 \times 10^{-3}} = 1400 \text{ Pa}$

$\Rightarrow \quad P_{in} = 1400 + 1.013 \times 10^5$

$$= 0.014 \times 10^5 + 1.013 \times 10^5 = 1.027 \times 10^5 \text{ Pa}$$

19. $h = \dfrac{2T \cos\theta}{rdg} \therefore h \propto \dfrac{1}{r}$. So the graph between h and r will be rectangular hyperbola.

20. Since water rises to height of 2 cm in a capillary



If tube is at 60°. In this case height must be equal to

$h = 2$ cm

$\Rightarrow \quad \cos 60° = \frac{h}{l}$

$\therefore \quad l = \frac{h}{\cos 60°} = \frac{2}{1/2} = 4$ cm

[CHEMISTRY]

21.

Lower the pK_a, stronger is the acid.

22.

$$\begin{array}{llccc} & HQ & \rightleftharpoons & H^+ & + \; Q^- \\ \text{At } t=0 & 0.1 & & - & - \\ \text{At Eqm.} & (0.1-x) & & x & x \end{array}$$

$pH = 3 \quad \Rightarrow \quad [H^+] = x = 10^{-3}$

$0.1 - x \simeq 0.1$

$K_a = \frac{x \cdot x}{0.1 - x} = \frac{10^{-3} \times 10^{-3}}{0.1} = \mathbf{10^{-5}}$

Alternatively :

$x = c\alpha = 10^{-3}$

$\alpha = \frac{10^{-3}}{0.1} = 10^{-2}$

$K_a = c\alpha^2 = 0.1 \times 10^{-4} = \mathbf{10^{-5}}$

23.

$[OH^-] = \frac{100 \times 0.2 - 100 \times 0.1}{1000} = 10^{-2}$

$\Rightarrow \quad pOH = 2 \quad \Rightarrow \quad pH = \mathbf{12}$

24.

Change of pH from 1 to 2 $\Rightarrow$ change in $[H^+]$ from 10^{-1} M to 10^{-2} M

$M_2V_2 (\text{dil.}) = M_1V_1 (\text{conc.})$

$V_2 = \frac{10^{-1} \times 1}{10^{-2}} = 10 L$

Volume of H_2O added = 10 – 1 = **9L**

25.

CH_3COOH is a weak acid. $10^{-2}M$ CH_3COOH will give much less $[H^+]$ concentration than 10^{-2} M. Hence, pH will be **more than 2.**

26.

$$pH = 3 \Rightarrow [H^+] = 10^{-3} M$$

On dilution, $[H^+] = \frac{1}{2} \times 10^{-3} = 5 \times 10^{-4} M$

New $pH = -\log(5 \times 10^{-4}) = -(0.699 - 4) = \mathbf{3.301}$

27.

$$\underset{}{MX_4} \rightleftharpoons \underset{S}{M} + \underset{4S}{4X}$$ (S is solubility in mol L^{-1})

$$K_{sp} = (S)(4S)^4 = 256 S^5$$

$$\boldsymbol{S = \left[\frac{K_{sp}}{256}\right]^{1/5}}$$

28.

$$MX_2 \rightleftharpoons \underset{S}{M^{2+}} + \underset{2S}{2X^-}$$ (S is solubility in mol L^{-1})

$$K_{sp} = S \times (2S)^2 = 4S^3$$

$$4S^3 = 4 \times 10^{-12}$$

$$S = \mathbf{1 \times 10^{-4}} \boldsymbol{M}$$

29.

$$K_{sp} \text{ of } Cr(OH)_3 = S \times 3^3 S^3$$

$$27S^4 = 1.6 \times 10^{-30}$$

$$S = \boldsymbol{\sqrt[4]{1.6 \times 10^{-30} / 27}}$$

30.

$$A_3B_2 \rightleftharpoons \underset{3x}{3A} + \underset{2x}{2B}$$ [for solubility of A_3B_2 as xM]

$$K_{sp} = [A]^3 \times [B]^2 = [3x]^3 \times [2x]^2 = \mathbf{108\, x^5}$$

31.

Solubility is directly proportional to K_{sp}. MnS has highest K_{sp} among the given substances and hence has highest solubility.

32.

$$CaF_2 \rightleftharpoons Ca^{2+} + 2F^-$$

For solubility 'S', $K_{sp} = (S)(2S)^2 = 4S^3$ (S is solubility)

$$4S^3 = 3.2 \times 10^{-11}$$

$$S^3 = 8 \times 10^{-12}$$

$$S = \mathbf{2 \times 10^{-4}} \boldsymbol{M}$$

33.

For solubility S, K_{sp} of $A_2B_3 = (2)^2 \times (3)^3 \times S^2 \times S^3 = 108 \times (1 \times 10^{-2})^5$

$$= 108 \times 10^{-10} = \mathbf{1.08 \times 10^{-8}}$$

34.

$$HgCl_2 \rightleftharpoons Hg^{2+} + 2Cl^-$$

$$K_{sp} = S \times (2S)^2 = 4S^3 \qquad (S \text{ is solubility})$$

$$4S^3 = 4 \times 10^{-15}$$

$$S = 10^{-5}$$

$$[Cl^-] = 2S = \mathbf{2 \times 10^{-5}\ M}$$

35.

Higher the K_{sp}, higher is the solubility.

36.

$$\underset{\text{For solubility 's'}}{PbCl_2} \rightleftharpoons \underset{s}{Pb^{2+}} + \underset{2s}{2Cl^-}$$

$$K_{sp} = 4s^3 \quad \Rightarrow \quad s = \left(\frac{1}{4} \times 10^{-6}\right)^{\frac{1}{3}}$$

$$= \mathbf{(0.25 \times 10^{-6})^{1/3}}$$

37.

If 1 L of each solution is mixed,

$$[H^+] = \frac{10^{-3} + 10^{-4} + 10^{-5}}{3}$$

$$= \frac{11.1 \times 10^{-4}}{3} = \mathbf{3.7 \times 10^{-4}\ M}$$

38.

$$\underset{\text{For solubility 's'}}{AgCl} \rightleftharpoons \underset{s}{Ag^+} + \underset{s}{Cl^-}$$

$$NaCl \longrightarrow \underset{0.1}{Na^+} + \underset{0.1}{Cl^-}$$

$$[Ag^+] + [Cl^-] = K_{sp} \Rightarrow \quad (s)(0.1) = 1.2 \times 10^{-10}$$

$$s = \mathbf{1.2 \times 10^{-9}\ M}$$

39.

40.

$$C\alpha^2 = K_a$$

$$[H^+] = C\alpha = \frac{K_a}{\alpha}$$

$$pH = -\log \frac{K_a}{\alpha}$$

$$= -\log K_a + \log \alpha$$

$$= -\log 10^{-9} + \log\left(\frac{0.01}{100}\right)$$

$$= +9 - 4 = \mathbf{5}$$

[MATHEMATICS]

41. (b) $\frac{2b^2}{a} = b \Rightarrow \frac{b}{a} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{4}$

Hence $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{3}}{2}$.

42 (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Since it passes through (–3, 1) and (2, –2), so $\frac{9}{a^2} + \frac{1}{b^2} = 1$ and $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4} \Rightarrow a^2 = \frac{32}{3}$, $b^2 = \frac{32}{5}$

Hence required equation of ellipse is $3x^2 + 5y^2 = 32$.

Trick : Since only equation $3x^2 + 5y^2 = 32$ passes through (–3, 1) and (2, –2). Hence the result.

43 (a) Given $\frac{2b^2}{a} = 10$ and $2b = 2ae$

Also $b^2 = a^2(1 - e^2) \Rightarrow e^2 = (1 - e^2) \Rightarrow e = \frac{1}{\sqrt{2}}$

$\Rightarrow b = \frac{a}{\sqrt{2}}$ or $b = 5\sqrt{2}$, $a = 10$

Hence equation of ellipse is $\frac{x^2}{(10)^2} + \frac{y^2}{(5\sqrt{2})^2} = 1$

i.e., $x^2 + 2y^2 = 100$.

44. (d) $e = \frac{1}{\sqrt{2}}$; Latus rectum $= \frac{2b^2}{a} = \frac{2a^2}{a}\left(1 - \frac{1}{2}\right) = a$

i.e., semi-major axis.

45. (a) Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\because$ It passes through (– 3, 1)

So, $\frac{9}{a^2} + \frac{1}{b^2} = 1 \Rightarrow 9 + \frac{a^2}{b^2} = a^2$ .....(i)

Given eccentricity is $\sqrt{2/5}$

So, $\frac{2}{5} = 1 - \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{3}{5}$ .....(ii)

From equation (i) and (ii), $a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$

Hence required equation of ellipse is $3x^2 + 5y^2 = 32$.

46. (b) $\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0 \Rightarrow \frac{x^2}{r-2} + \frac{y^2}{5-r} = 1$

Hence $r > 2$ and $r < 5 \Rightarrow 2 < r < 5$.

47. (a) The ellipse is $4(x-1)^2 + 9(y-2)^2 = 36$

Therefore, latus rectum $= \frac{2b^2}{a} = \frac{2.4}{3} = \frac{8}{3}$.

48. (b) $4x^2-8x+y^2+2y+1=0$

$\Rightarrow (2x-2)^2+(y+1)^2=-1+4+1$

$\Rightarrow \frac{(x-1)^2}{1}+\frac{(y+1)^2}{4}=1 \Rightarrow e=\sqrt{1-\frac{1}{4}} \Rightarrow e=\frac{\sqrt{3}}{2}$.

49. (a) Let any point on it be (x,y), then $\frac{\sqrt{(x+1)^2}+\sqrt{(y-1)^2}}{\left|\frac{x-y+3}{\sqrt{2}}\right|}=\frac{1}{2}$

Squaring and simplifying, we get

$7x^2+2xy+7y^2+10x-10y+7=0$.

50. (b) $c=\pm\sqrt{b^2+a^2m^2}=\pm\sqrt{4+8.4}=\pm 6$.

51. (b) $\frac{2b^2}{a}=b \Rightarrow \frac{b}{a}=\frac{1}{2} \Rightarrow \frac{b^2}{a^2}=\frac{1}{4}$

Hence $e=\sqrt{1-\frac{b^2}{a^2}}=\frac{\sqrt{3}}{2}$.

52. (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Since it passes through (–3, 1) and (2, –2),

so $\frac{9}{a^2}+\frac{1}{b^2}=1$ and $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4} \Rightarrow a^2=\frac{32}{3}$, $b^2=\frac{32}{5}$

Hence required equation of ellipse is $3x^2+5y^2=32$.

Trick : Since only equation $3x^2+5y^2=32$ passes through (–3, 1) and (2, –2). Hence the result.

53. (c) $\frac{x^2}{\frac{112}{16}}+\frac{y^2}{\frac{112}{7}}=1$. Therefore, $e=\sqrt{1-\frac{112}{16}.\frac{7}{112}}=\frac{3}{4}$.

54. (b) Here given that $2b=10, 2a=8 \Rightarrow b=5,\ a=4$

Hence the required equation is $\frac{x^2}{16}+\frac{y^2}{25}=1$.

55. (c) Let point be (h,k) their pair of tangent will be

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}-1\right)=\left(\frac{hx}{a^2}+\frac{yk}{b^2}-1\right)^2$$

Pair of tangents will be perpendicular, if

coefficient of x^2 + coefficient of $y^2=0$

$\Rightarrow \frac{k^2}{a^2b^2}+\frac{h^2}{a^2b^2}=\frac{1}{a^2}+\frac{1}{b^2} \Rightarrow h^2+k^2=a^2+b^2$

Replace (h,k) by $(x,y) \Rightarrow x^2+y^2=a^2+b^2$.

56. (c) Focal distance of any point P (x,y) on the ellipse is equal to $SP=a+ex$. Here $x=a\cos\theta$

Here $SP=a+ae\cos\theta=a(1+e\cos\theta)$.

57. (a) Let point $P\ (x_1, y_1)$

So, $\sqrt{(x_1+2)^2+y_1^2}=\frac{2}{3}\left(x_1+\frac{9}{2}\right)$

$\Rightarrow (x_1+2)^2+y_1^2=\frac{4}{9}\left(x_1+\frac{9}{2}\right)^2$

$\Rightarrow 9[x_1^2+y_1^2+4x_1+4]=4\left(x_1^2+\frac{81}{4}+9x_1\right)$

$\Rightarrow 5x_1^2+9y_1^2=45 \Rightarrow \frac{x_1^2}{9}+\frac{y_1^2}{5}=1$,

Locus of (x_1, y_1) is $\frac{x^2}{9}+\frac{y^2}{5}=1$, which is equation of an ellipse.

58. (b) $SP+S'P=2a=2.6=12$.

59. (c) In the first case, eccentricity $e=\sqrt{1-(25/169)}$

In the second case, $e'=\sqrt{1-(b^2/a^2)}$

According to the given condition,

$\sqrt{1-b^2/a^2}=\sqrt{1-(25/169)}$

$\Rightarrow b/a=5/13$, $(\because a>0, b>0)$

$\Rightarrow a/b=13/5$.

60. (b) Foci $=(3,-3) \Rightarrow ae=3-2=1$

Vertex $=(4,-3) \Rightarrow a=4-2=2 \Rightarrow e=\frac{1}{2}$

$\Rightarrow b=a\sqrt{\left(1-\frac{1}{4}\right)}=\frac{2}{2}\sqrt{3}=\sqrt{3}$

Therefore, equation of ellipse with centre $(2,-3)$ is

$\frac{(x-2)^2}{4}+\frac{(y+3)^2}{3}=1$.